| Course Type | Course Code | Name of the Course | L | Т | P | Credits |
|----------------|-------------|-------------------------------------|---|---|---|---------|
| DE | NMCD543 | Introduction to Commutative Algebra | 3 | 0 | 0 | 3 |

Course Objective

This is an introductory course in commutative algebra exploring the theory of commutative rings, modules, and ideals. This course is essential for anyone who wants to do research in areas such as commutative algebra, algebraic geometry, and algebraic number theory, while developing proof-writing skills.

Learning Outcomes

Students will gain proficiency in working with commutative rings, modules, and ideals. They will be able to apply these concepts to solve problems in algebraic geometry, algebraic number theory, and related fields, while strengthening their ability to construct rigorous mathematical proofs.

| Unit No. | Topics to be Covered | Lecture Hours | Learning Outcome | |
|-------------|--|------------------|---|--|
| 1 | Review of prime and maximal ideals, nilradical and Jacobson radical, Operations on ideals, extension and contraction | 5 | The main outcome of this unit is to develop the idea of nilradical, Jacobson radical and the extension and contraction of ideals. | |
| 2 | Modules, submodules, quotient modules and module homomorphisms. Generation of modules, direct sum and product, Exact sequences | 10 | The main outcome of this unit is to develop the idea of modules. | |
| | Tensor products of modules, Restriction and extension of scalars, exactness properties of the tensor product | 5 | | |
| 3 | Finitely generated modules over principal ideal domains, invariant factors, elementary divisors, rational canonical forms. Applications to finitely generated abelian groups and linear transformations. | 10 | The main outcome of this unit is to develop the idea of finitely generated modules over PIDs. | |
| 4 | Rings and Modules of fractions, Integral dependance, Going Up and Going Down theorems | 6 | The main outcome of this unit is to develop the idea of localization and integral dependance. | |
| 5 | Noetherian rings and modules, Primary decomposition in Noetherian rings, Artin rings | 6 | The main outcome of this unit is to develop the idea of Noetherian and Artin rings. | |
| | Total | 42 | , | |

Text Books:

- 1. M. F. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra, Addison Wesley, 1969.
- 2. D. Eisenbud, Commutative Algebra (with a view toward algebraic geometry), Graduate Texts in Mathematics 150, Springer-Verlag, 2003.

Reference Books:

- 1. N. Jacobson, Basic Algebra I and II, 2nd Edition, W. H. Freeman, 1985 and 1989.
- 2. S. Lang, Algebra, 3rd Edition, Springer (India), 2004.
- 3. O. Zariski and P. Samuel, Commutative Algebra, Vol. I, Springer, 1975.